2) Simplicity. The control logic cited in Eq. (28) is extremely simple, and the results of this study indicate that the servomechanism requirements for implementation of the system are modest, both in terms of peak force and frequency response requirements. Sensor requirements are also modest, since only w and ω_y must be measured. The implementation of this system would be considerably simpler than competitive systems such as fluid-displacement systems, movable mass on boom arrangements, inertia wheels, CMG's, etc.

3) Power Requirements. The low level of peak and sustained force levels required for the controller indicates a quite low level of power consumption. A movable mass controller has a significant advantage, in this respect, over CMG's or inertia wheels.

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Attitude Stabilization of Synchronous Communications Satellites Employing Narrow-Beam Antennas

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Future communications satellites will employ multiple narrow-beam antennas covering widely separated areas of the Earth. Some of these narrow beams must cover more than one ground station, necessitating coverage of stations located at the fringes of the antenna beams. This imposes a requirement for high pointing accuracies; current estimates for satellites to be put into service in 1972 and beyond being for beam pointing accuracies of 0.1° or less. An additional requirement for future satellites will be longer life-times; operating lives of 10 yr are presently being considered. Attitude stabilization systems using momentum and reaction wheels in various combinations are evaluated based on these requirements. The candidate wheel stabilization systems are discussed in terms of their respective design parameters and performance characteristics. Comparisons are made between the systems on the basis of wheel size, weight, and power required to achieve high pointing accuracies.

Nomenclature

B = antenna beam width
E = antenna elevation angle in radians
H_z = yaw wheel momentum
h = angular momentum of pitch wheel
h_R = required control momentum
I_x,I_y,I_z = principal moments of inertia
K = autopilot gain
$K_{\dot\theta},K_\theta,K_I$ = controller rate, position and integral gain, respectively
K_D = desaturation gain
k = factor relating required momentum to peak disturbance torque, or ratio of yaw gain to roll gain
M_{xc},M_{zc} = control moments about roll and yaw axes, respectively
N = correction factor
n = number of days between momentum desaturation
P_P,P_A = peak and average power, respectively
P_m,P_R = momentum wheel power and three-axis reaction wheel system power, respectively
s = Laplace operator
T_c = cyclic disturbance torque
T_M,T_{MP} = motor torque for disturbances and peak motor torque, respectively
T_p = peak disturbance torque
T_S = inertial fixed disturbance torque
T_x,T_z = disturbance torques about roll and yaw axes, respectively
W = weight of momentum control system
α = offset angle of roll-yaw coupled control thruster
ϵ = attitude pointing error
ϕ = roll angle
ψ = yaw angle
θ = pitch angle
τ = lead time constant in control system
τ_m = motor time constant
τ_z = nonminimum phase zero time constant
ω_0 = orbital rate, 7.29×10^{-5} rad/sec for synchronous orbit

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$\omega_1, \omega_2, \omega_3$ = frequency of roots of closed loop control system
$\zeta_1, \zeta_2, \zeta_3$ = damping factors associated with closed loop frequency roots

Subscripts

m = momentum wheel
s = sensor
ss = steady state

Introduction

INTERNATIONAL communications satellites launched to date have been spin-stabilized, either with or without despun antennas. Because of increased electrical power requirements for the increased communications capacity of future satellites and the limitations on drum size imposed by launch vehicle constraints, it may no longer be possible to mount a sufficient number of solar cells on a spinning drum. To meet this power requirement, it is expected that a three-axis stabilized satellite with sun tracking solar arrays will be a prime candidate for future missions. Since there may be several antennas, each pointing at a different area of the Earth, it is not practical to maneuver the main body of the spacecraft to accomplish pointing. The antennas may be steered open loop with respect to the main body, which will be stabilized such that its yaw axis is aligned with the local vertical.

For a long-life, synchronous communication satellite, the class of stabilization systems using momentum and reaction wheels in various combinations as control torque sources are considered the best choices on the basis of applicability, complexity, life, and proven performance. A three-axis stabilized system will require a long-life sensor package for providing accurate, Earth-referenced pointing information. The candidate stabilization systems can be subdivided into two subclasses based on sensor requirements; those requiring a yaw sensor and those which do not. A system that does not require a yaw sensor depends on the gyroscopic stiffness, which is due to a momentum wheel nominally aligned normal to the orbit plane, to provide the yaw pointing accuracy.

The effect of pointing accuracy on communications system performance is shown in Fig. 1, which shows the change in antenna gain for ground stations initially at boresight and those on the half-power contour as a function of normalized pointing error. The attitude pointing error is normalized by the antenna beam width. Although the gain improvement due to more accurate pointing is small at boresight, the improvement at the half power point is appreciable. The attitude accuracy of present communications satellites is 0.35°. If the antenna beam width is 1°, a 4.4 db gain is obtained at the half power point by reducing the pointing error from 0.35° to 0.1°. In the case of multiple antennas, the improvement is dependent on over-all mission considerations, but it is to be expected that large benefits would result from improved pointing accuracy.

The pointing accuracy is determined by the attitude stabilization accuracy and antenna misalignments, each of which is essentially an independent design quantity. Antenna misalignment can be minimized by accurate mechanical alignment procedures and proper thermal and mechanical design. Procedures and methods for minimizing antenna alignment will not be considered but are recognized as being an integral part of any precision pointing system. Values for alignment error may vary from less than 0.01° to 0.07°, depending on the antenna design and alignment procedures used. The primary consideration of the paper will be the impact of the attitude stabilization system on beam pointing accuracy.

The evaluation and comparison of attitude stabilization systems in the beam pointing mode of operation is presented in the following sections. The candidate stabilization systems are discussed in terms of their respective control system design parameters and performance characteristics. Preliminary design comparisons are made between candidate systems based on wheel size and attitude accuracy.

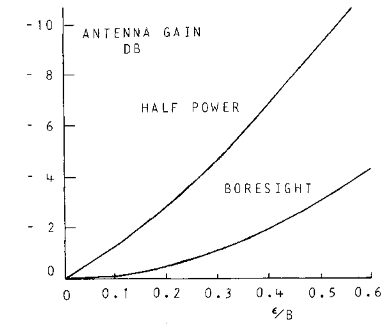


Fig. 1 Antenna gain vs pointing error.

Attitude stabilization systems employing one, two, and three wheels are to be considered. The characteristics of each system will be explained in terms of the linearized analysis of the system. In all systems, pitch axis control is provided by a reaction wheel, or momentum wheel whose speed can be varied. The pitch control system design concept is conventional and is deferred until the discussion of the three-wheel system.

In the one- and two-wheel systems, a yaw sensor is not used. The feature differentiating between these systems is the different method of obtaining yaw stability. Yaw restraint is provided in the one- and two-wheel systems by utilizing a large angular momentum wheel nominally aligned normal to the orbit plane. The discussion of the different momentum wheel concepts will be restricted to the roll-yaw control channels. The concepts will be explained in terms of the linearized system equations. The emphasis is on the functional form of the roll and yaw control signals. Hardware considerations are incorporated in the system discussion once the fundamental principle of operation is described. The discussion will progress from system to system in a building-block approach to add continuity to the presentation, but each system could be viewed as an independent concept distinct from the others.

Wheel Control System Concepts

The one-wheel system[1] is an active, three-axis attitude control system incorporating a momentum wheel and mass expulsion jets. The wheel is nominally aligned normal to the orbit plane. The system is shown in Fig. 2. Roll error signals from the attitude sensor are processed by a lead controller, which is best implemented by a pseudorate circuit, and used to fire gas valves to supply control torques simultaneously about the roll and yaw axes. Restraint about the yaw axis is provided by the momentum wheel with its angular momentum vector aligned along the negative pitch axis.

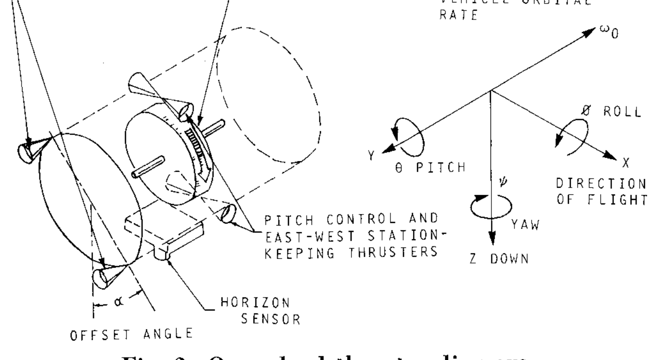


Fig. 2 One-wheel thruster diagram.

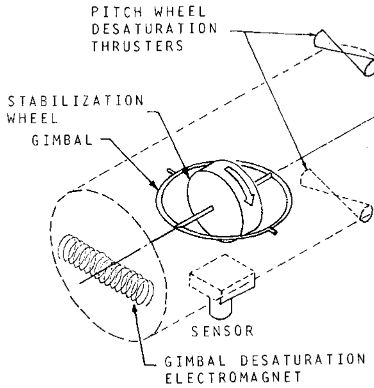


Fig. 3 Single gimballed one-wheel system diagram.

Damping of the system is provided in the roll channel by the pseudorate controller which provides a lead effect. In particular, the controller in the roll channel serves to damp the nutation frequency mode of the coupled roll-yaw vehicle dynamics. System damping is provided in yaw by offsetting the roll valves such that they supply control torques about the yaw axis. The roll thruster offset into yaw is designed to damp the orbit frequency mode associated with the vehicle dynamics. The unique feature of the one-wheel system is the use of the offset roll-actuated control torque and the momentum wheel to control the yaw axis without a direct yaw sensor. The linearized roll and yaw dynamics and kinematic equations for the system are given by

$$\begin{bmatrix} I_x s^2 + \omega_0 h & +hs \\ -hs & I_z s^2 + \omega_0 h \end{bmatrix} \begin{bmatrix} \phi(s) \\ \psi(s) \end{bmatrix} = \begin{bmatrix} M_{x_c} + T_x \\ M_{z_c} + T_z \end{bmatrix} \quad (1)$$

The functional forms of the roll and yaw control torques for the one-wheel system with thrusters are

$$\begin{aligned} M_{x_c} &= -K \cos\alpha(\tau_s + 1)\phi(s) \\ M_{z_c} &= K \sin\alpha(\tau_s + 1)\phi(s) \end{aligned} \quad (2)$$

The gain K is determined by the thruster force, moment arm, and linear range on the controller. Inserting these control torques into the linearized dynamics equations and factoring the resulting fourth-order characteristic equation into high-and low-frequency roots yields two quadratic equations with associated damping ratio and natural frequency:

$$\begin{aligned} \omega_1 &= (K \cos\alpha/NI_x)^{1/2} \quad \zeta_1 = (NK \cos\alpha/I_x)^{1/2}\tau/2 \\ \omega_2 &= (N\omega_0 h/I_z)^{1/2} \quad \zeta_2 = (Nh/I_z\omega_0)^{1/2} \tan\alpha/2 \end{aligned} \quad (3)$$

where

$$N = 1/(1 + h^2/I_z K \cos\alpha) \quad (4)$$

The term defined in Eq. (4) can be considered a correction factor with a nominal value of unity. For systems using a large momentum wheel, the correction factor should be calculated and used in the approximations for natural frequency and damping ratio given by Eq. (3). The above equations are based upon the assumption that the following conditions hold:

$$\begin{aligned} h \gg \max[&4\omega_0(I_y - I_z), \omega_0(I_x - I_y + I_z), \\ &\omega_0(I_y - I_x), \tau\omega_0/\tan\alpha] \end{aligned} \quad (5)$$

$$K \cos\alpha \gg 4\omega_0 h I_x/I_z$$

The steady-state yaw offset error, due to a constant orbit referenced torque is given by

$$\psi_{ss} = 57.3(T_z - T_x \tan\alpha)/\omega_0 h \quad (6)$$

Typically, the offset angle α is only a few degrees, and consequently the roll torque contribution is much less than the yaw torque effects and can be neglected for preliminary design studies. Thus, the peak yaw angle due to an external torque is given approximately by

$$\psi \approx 57.3T_z/\omega_0 h \quad (7)$$

Equation (7) is used to size the momentum wheel based on an estimate of yaw torque and allowable yaw offset. The derivation of Eq. (7) is based on the assumption that the external torque is constant in the orbit frame. Under this condition, the equation is applicable for sizing all momentum wheel systems and will not be rederived for the succeeding wheel systems. Equation (7) is also useful for estimating the amplitude for yaw oscillations caused by a secular torque for both the one-wheel with thrusters system and the double gimballed wheel system.

In any system using thrusters, the number of thruster cycles in a 10-yr life are a potentially limiting item. Also, the wheel system will tend to exhibit a nutation when within the thruster deadbands. The amplitude of this nutation is dependent on the minimum impulse bit from the thruster; reducing the impulse bit to decrease the nutation amplitude will increase thruster cycles.

Single Gimballed One-Wheel and Two-Wheel Systems

The single gimballed one-wheel system uses a controller to shape the roll error signal and drive the gimbal angle. The gimbal axis is aligned along the roll axis and its null position is such that the spin axis of the momentum wheel is aligned along the negative pitch axis. Rotating the gimbal from null produces a component of angular momentum along the z axis. The system is shown in Fig. 3. To stabilize the orbit rate roots, some desaturation mechanism is needed. This mechanism could be an electromagnetic torque, a mass expulsion system, or perhaps, gravity gradient. The control signal to drive the magnet or thrusters could be derived from the gimbal angle.

An alternate dynamical mechanization would be to use two wheels. One wheel, a large momentum wheel providing gyroscopic stiffness; the other, a small reaction wheel with its spin axis aligned along the z axis. This system is shown in Fig. 4.

The roll error signal is shaped and used to vary either the speed of the yaw wheel or the gimbal angle. A rather unusual form of shaping, or compensation (i.e., a right half plane zero), has been suggested[2] because it requires the least hardware to provide adequate damping of the high-frequency coupled roll and yaw motion. The discussion of the system compensation is given in terms of the two-wheel system. The linearized equations of the coupled roll and yaw dynamics for the system are

$$\begin{bmatrix} I_x s^2 + \omega_0 h & hs & -\omega_0 \\ -hs & I_z s^2 + \omega_0 h & K_D + s \\ K(1 - \tau_z s) & 0 & s(\tau_M s + 1) \end{bmatrix} \times \begin{bmatrix} \phi(s) \\ \psi(s) \\ H_z(s) \end{bmatrix} = \begin{bmatrix} T_x(s) \\ T_z(s) \\ 0 \end{bmatrix} \quad (8)$$

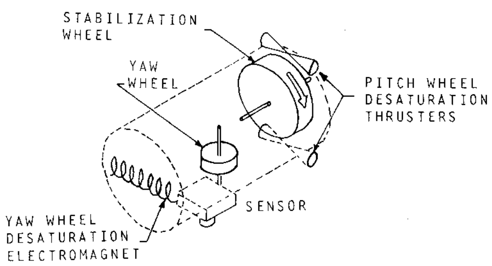


Fig. 4 Two-wheel system diagram.

The characteristic equation of the system is of the sixth order and can be factored into three pairs of roots with natural frequencies and damping ratios given approximately by

$$\omega_1 \approx h/(I_xI_z)^{1/2} \qquad \zeta_1 \approx (\tau_z\omega_1/2)/(\omega_1^2/\omega_2^2 - 1)$$
$$\omega_2 \approx K/h\tau_M \qquad \zeta_2 \approx (\tfrac{1}{2}\omega_2\tau_M)(\omega_1^2/\omega_2^2 - \omega_1^2\tau_z\tau_M - 1)/(\omega_1^2/\omega_2^2 - 1) \quad (9)$$
$$\omega_3 \approx \omega_0 \qquad \zeta_3 \approx K_D/2\omega_0$$

Analysis has indicated that the most rapid damping of transients is obtained if $\zeta_1 \approx 0.175$, $\zeta_2 \approx 0.707$, and $\zeta_1\omega_1 \approx \zeta_2\omega_2$. These three relations allow K, τ_z, and τ_M to be selected. The value of h is determined by the maximum allowable yaw offset and is found from Eq. (7).

The damping of the orbit rate root is directly dependent upon the desaturation gain K_D, whose magnitude will be limited by weight and/or power constraints. This additional damping which is required for the orbit rate roots could be provided by using an electromagnet torquer to supply desaturating torques to the yaw reaction wheel. Interaction of this electromagnet with the Earth's magnetic field (approximately 100 γ at synchronous altitude) would produce small (i.e., order of 10^{-5} ft-lb) body torques about the yaw axis. A magnetic moment of about 10^{-5} unit-pole-cm is required which corresponds to an electromagnet weighing 3–6 lb and using 1–2 w of power.

The desaturation mechanism could be eliminated if two gimbal axes were used, as discussed in the next system.

Double Gimballed One-Wheel System

The basic components of the double gimballed one-wheel system[3] are a roll sensor signal, a compensation network, a momentum wheel mounted in two-degree-of-freedom gimbals, and a decoupling computer. Roll and yaw control torques are provided by driving the roll and yaw gimbals. The system is shown in Fig. 5.

The dynamic behavior of the double gimballed wheel system would be very similar to that of the one-wheel with thrusters system except that the double gimballed system incorporates nutation and orbit rate decoupling. The nutation motion is decoupled in roll by subtracting the roll error from the roll gimbal drive signal. Physically, this allows the vehicle to roll while the momentum wheels orientation remains fixed in inertial space. This feature significantly reduces yaw transients due to roll transients. One method for achieving the orbit rate decoupling is a clock-driven resolver-integrator-resolver combination. The first resolver transforms the coupled roll-yaw reaction control torque command signal to an inertial frame. Then it is integrated to form a momentum command signal which is transformed by the second resolver back into the body frame as a momentum command (i.e., gimbal angle command).

Roll attitude is controlled by applying a reaction torque to the roll gimbal in response to an error from the attitude sensor. The roll rate and attitude gains are determined by the parameter values in the compensation network. Yaw is controlled by a gyrocompassing technique similar to that of the one-wheel with thrusters system. As in that system, the yaw attitude gain is determined by the amount of gyroscopic coupling, i.e., the product $h\omega_0$. The yaw rate gain is produced by producing a small yaw torque whenever a roll torque is commanded. In the one-wheel with thrusters system, the yaw rate gain is produced by off-setting the roll thrusters into yaw by an angle α. In the double gimballed one-wheel system, the yaw rate gain is produced by using a fraction of the roll gimball command signal to drive the yaw gimbal.

The linearized equations of the roll-yaw dynamics for the

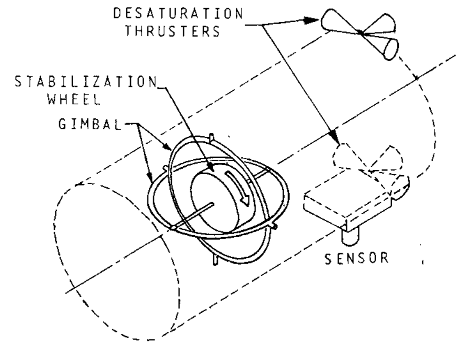


Fig. 5 Double gimballed one-wheel system diagram.

double gimballed one-wheel system are

$$\begin{bmatrix} I_xs^2 + K(\tau s + 1) & hs \\ -kK(\tau s + 1) & I_zs^2 + \omega_0 h \end{bmatrix} \begin{bmatrix} \phi(s) \\ \psi(s) \end{bmatrix} = \begin{bmatrix} T_x(s) \\ T_z(s) \end{bmatrix} \quad (10)$$

The resulting fourth-order characteristic equation can be factored into two pairs of roots with natural frequencies and damping ratios given approximately by

$$\omega_1 \approx (K/I_x)^{1/2} \qquad \zeta_1 \approx (K/I_x)^{1/2}\tau/2$$
$$\omega_2 \approx (\omega_0 h/I_z)^{1/2} \qquad \zeta_2 \approx (h/\omega_0 I_z)^{1/2}k/2 \quad (11)$$

The roll autopilot gain is selected large enough to insure capture capability and rapid roll transient response. The lead time constant is selected to provide critical damping of the high-frequency root. The ratio of yaw gain to roll gain is chosen so that the low-frequency root is also critically damped.

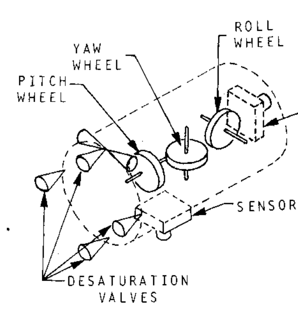
The double gimballed wheel could also be driven by a signal from a yaw sensor allowing a decoupling of the roll and yaw motion. Then the yaw offset would no longer be a function of gyroscopic stiffness provided by the momentum wheel. The roll and yaw gimbals could be driven by control laws of the same type as those for the reaction wheels of the three-axis system presented in the next section.

Three-Axis Reaction Wheel System

The primary components of a three-axis reaction wheel control system[4] are a sensor package supplying three-axis attitude information, the operational controller, the reaction wheels, and the desaturation thrusters. The pitch wheel in the three-wheel system is not used to provide gyroscopic yaw restraint. In its most basic form, the three-wheel system can be considered as three parallel pitch, roll and yaw control systems, independently controlling each axis by varying the speed of a reaction wheel in response to the attitude error about that axis. The system is shown in Fig. 6.

Each compensated attitude signal drives a torque motor to vary the speed of the reaction wheel. The controller compensation consists of an integral term in addition to the conventional proportional and possibly rate terms. The integral

Fig. 6 Three-axis reaction wheel system diagram.



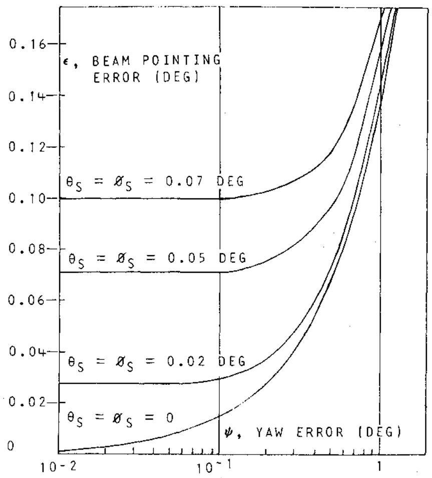


Fig. 7 Pointing error vs yaw off-set.

term is an accumulation of attitude error and it serves to minimize spacecraft off-set associated with external torques or internal momentum transfer. The roll and yaw channels are coupled through the vehicle dynamics, and there will consequently be a continuous transfer of momentum between the roll and yaw wheels.

The characteristic equation for any one axis will be representative of the other axes as well since the controller for each axis is of the same form and dynamic cross coupling is small. The characteristic equation for the pitch channel is

$$\tau_M I_y s^3 + (I_y + K_{\dot{\theta}})s^2 + K_\theta s + K_I = 0 \qquad (12)$$

For a small integrator gain, the dynamic behavior is similar to a second-order system with natural frequency and damping ratio given by

$$\omega_N = (K_\theta/\tau_M I_y)^{1/2} \qquad \zeta = (I_y + K_{\dot{\theta}})(4\tau_M I_y K_\theta)^{1/2} \qquad (13)$$

In addition to this pair of complex roots, the system has a small negative real root at approximately $-K_I/K_\theta$.

Discussion of Wheel Concepts

The accuracy achievable with each of the systems is fundamentally sensor-limited. For the class of system which does not utilize a yaw sensor, the accuracy is also dependent on the angular momentum of the momentum wheel. From a purely analytic viewpoint, parameter values can be chosen such that each system could meet the beam pointing accuracy requirements. In some cases, the set of parameters which must be chosen to meet pointing requirements may be unrealizable based on hardware considerations or power requirements.

The basis on which the systems are evaluated must be physical realizability and ability to meet, or be compatible with, the various modes of attitude stabilization. In the following sections, wheel sizing, weight, power, sensor considerations, and compatibility with potential attitude stabilization modes are discussed as they relate to the wheel systems. Basically, the comparisons will be in terms of a momentum wheel system and a three-axis reaction wheel system.

Attitude Stabilization System Beam Pointing Accuracy

The beam pointing error ϵ is shown in Fig. 7 as a function of yaw error ψ for equal fixed values of pitch and roll sensor errors. The curves are for a synchronous equatorial satellite with its antenna boresight normally aligned to approximately 60° latitude. The effect of decreasing the latitude is to shift the curves to the right. For this simple, yet practical, case it is observed that for yaw errors less than 0.1°, beam pointing error is dominated by the pitch and roll sensor errors. The beam pointing error in Fig. 7 is described by

$$\epsilon = [\theta_s^2 + (E\psi)^2 + \phi_s^2]^{1/2} \qquad (14)$$

The roll, pitch, and yaw errors have been assumed to be statistically independent. The effects of antenna misalignment can be considered by statistically eliminating the antenna error from the total beam pointing error to obtain that portion of the beam pointing error allotted to the attitude stabilization system.

For a system without a yaw sensor, the yaw error in Fig. 7 may be interpreted as the peak yaw angle resulting from environmental torques as given in Eq. (7). For a system with a yaw sensor, the error is the yaw sensor error. As will be shown in the following sections, the yaw error is an important parameter in the comparison of these two classes of control systems.

Wheel Momentum, Weight and Power Relationships

The ratio of momentum wheel to reaction angular momentum, and the ratio of wheel weights will be derived. Equation (7) gives a relationship for yaw angle as a function of angular momentum and yaw torque. Rewriting this expression, solving for wheel momentum gives

$$h_m = 57.3T_z/\omega_0\psi \qquad (15)$$

where the subscript m has been used to designate momentum wheel.

The angular momentum of the reaction wheel is a function of external torque and frequency of momentum dumping. A conservative estimate of the required angular momentum per axis is given by

$$h_R = T_c/\omega_0 + 8.64 \times 10^4 T_s n \qquad (16)$$

where T_c and T_s are the magnitude of the cyclic and inertially fixed torque on a specific control axis. The frequency of desaturation in days is given by n, and ω_0 is orbit rate.

Writing Eq. (16) in terms of peak torque on an axis, i.e., $T_p = T_c + T_s$, and the ratio of inertially fixed to peak torque, obtains

$$h_R = (1 - T_s/T_p + 6.3nT_s/T_p)T_p/\omega_0 \qquad (17)$$

or

$$h_R = kT_p/\omega_0 \quad \text{with} \quad k = (1 - T_s/T_p + 6.3nT_s/T_p) \qquad (18)$$

If T_p is interpreted as the largest peak torque on the spacecraft and all reaction wheels are to be identical, then the total required momentum is $3h_R$.

A relationship for the ratio of angular momentum can be derived from Eqs. (15) and (18) if T_z and T_p are assumed to be equal. For the initial design phase, this is a good approximation;

$$h_m/3h_R = 57.3/3k\psi \qquad (19)$$

The angular momentum ratio is inversely proportional to the desired yaw angle; hence, the more accurate the beam pointing, the smaller the yaw angle and the larger the ratio.

A rule-of-thumb relationship of wheel housing and associated electronics weight to the angular momentum for both reaction wheels and momentum wheels is

$$W = 7h^{0.4} \qquad (20)$$

The relationship is based on a fit to published vendor data. It agrees well with other published results.[5]

The ratio of system weights based on the wheels alone is obtained by substituting Eq. (19) in Eq. (20):

wt of momentum wheel/wt of reaction wheel =

$$1.7/(k\psi)^{0.4} \quad (21)$$

A comparison can be made of the ratio of the weight of a momentum wheel to a reaction wheel system as a function of the desired beam pointing error. As shown in Fig. 7, beam pointing error can be related to the peak yaw error. A relationship can be derived from the ratio of wheel weights assuming the yaw torque and peak torques on both systems are equal. The ratio is defined in Eq. (21). The factor k from Eq. (18) is a function of the number of days between momentum desaturation and the ratio of inertially fixed to peak torque. Values of k lie in the range of 1–40. A value of $k = 1$ corresponds to momentum desaturation every day, $k = 10$, to desaturation at most once a week, and $k = 40$, to desaturation at most once a month. The conclusions based on the values of k of 10 and 40 depend on the assumption that at most 20% of the torque is inertially fixed.

The ratio of wheel weights based on Eq. (21) is shown by Fig. 8. For yaw angles of less than 0.1°, the momentum wheel is always heavier than a three-axis reaction wheel package. The more frequent the momentum desaturation, the greater the reaction wheel weight advantage. Other factors, such as number of cycles on the mass expulsion system, tend to recommend a value of $k = 10$ for use in preliminary system tradeoff comparisons.

A desired bound in beam pointing error and an estimate of sensor error dictates a bound on yaw error. This yaw error will give a weight ratio. Figure 7 indicates for a 0.1° beam pointing error the range in allowable peak yaw angle is from 0.1° to 0.7°, depending on sensor error. The wheel weight ratio from Fig. 8 indicates the range on reaction wheel weight advantage is from 0.85 to 1.75 for $k = 10$ and from 2 to 4.3 for $k = 1$. In general, for precision beam pointing, a reaction wheel system can be expected to have a potential weight advantage.

The power required for a momentum and reaction wheel system is based on consideration of vendor data and Ref. 5. Approximations to peak power and average power for the reaction wheel are

$$P_p = 250T_{MP} \quad (22)$$

$$P_A = 1.2T_M + 1.5 + 0.008h \quad (23)$$

The equations assume a peak wheel speed in the range of 2000–3000 rpm and a 90% motor efficiency. The electronics are assumed 80% efficient with a standby power of 0.5 w. The running power for the wheel is given by the expression $(1 + 0.008h)$. Except during start-up, the average power for the momentum wheel system is essentially equal to that of a reaction wheel for which T_M is less than 0.1 ft-lb.

In the beam pointing mode, the ratio of momentum wheel power to three-axis reaction wheel system power is approximately

$$P_M/P_R = 0.66 + 0.18 \times 10^{-2}h \quad (24)$$

The reaction wheel system power is less than that of the momentum wheel when h is greater than about 185 ft-lb-sec.

Angular Momentum Sizing of Reaction and Momentum Wheel

Wheel sizing is primarily determined by the magnitude of the environmental torque. At synchronous altitude, the dominant environmental torque is due to solar radiation pressure unbalance. A reaction wheel system will reach momentum saturation due to inertially fixed components of solar torque. A three-axis stabilized communications satellite is referenced to a locally vertical Earth-oriented control axis. With respect to this rotating control reference frame an inertially fixed component of solar torque appears as a secular pitch torque and/or as an orbit rate cyclic component of roll and yaw torque. Hence, constant pitch torques or orbit rate cyclic roll and yaw torques cause momentum saturation of the reaction wheels. The wheel is desaturated by use of a mass expulsion system. The value of the angular momentum of a reaction wheel is based on the desired frequency of momentum desaturation and the size of inertial components of solar torque which are to be absorbed by the wheel.

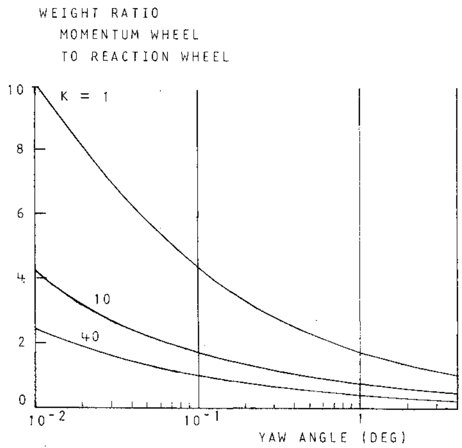


Fig. 8 Weight ratio of momentum and reaction wheels.

Equation (17) is used to estimate reaction wheel size based on a knowledge of disturbance torque magnitudes and frequency of momentum desaturation. The factor k is a function of the number of days between momentum desaturation and the ratio of inertially fixed to peak torque. The definition of k is given by Eq. (18). As an example, consider a case where the peak torque is 10^{-4} ft-lb, 10% of which is inertially fixed, and the wheel is to be desaturated every week. The factor k is found from Eq. (18) to be $k = 5.3$ and the required angular momentum per axis is found to be approximately 7.3 ft-lb-sec.

Consider sizing a momentum wheel for a system with a sensor which has a pitch and roll error of 0.05° and the desired beam pointing accuracy is 0.1°. Figure 7 indicates that a yaw angle of 0.5° or less is needed to meet pointing requirements. Equation (7) indicates that an angular momentum of at least 160 ft-lb-sec is needed to limit the peak yaw angle to 0.5° in the presence of a peak yaw component of solar torque of 10^{-4} ft-lb.

Estimates of wheel size depend on preliminary knowledge of the solar torque on the spacecraft, which is dependent on the specific spacecraft configuration and size. For a wide class of missions, the peak solar torque is in the range of 10^{-5} to 10^{-4} ft-lb, and typically, the cyclic torques dominate. Large unfurlable antennas, 20 ft or more in diameter, and a particularly unbalanced configuration could lead to peak torques above 10^{-4} ft-lb. The wheel size can be significantly reduced by designing a configuration which minimizes the solar torque. In the case of systems using gyroscopic stiffness to limit yaw, a particular premium is placed on minimizing the yaw torque. As a first approximation in wheel system comparisons, it is assumed that the peak torque occurs in yaw, although it must be realized that a particular design may be chosen to minimize yaw torques.

Sensors

The capability for precise three-axis Earth pointing stabilization depends on minimization of sensor errors. The sensors and associated processing, whether autonomous in the satellite or performed on the ground, must give attitude signals relative to a locally vertical Earth reference system. Vertical errors in pitch and roll must be less than 0.07° to have 0.1° beam pointing accuracy. Earth pointing sensors are a logical candidate for onboard measurements. Direct vertical references can come from electronic processing of detected Earth radiations, either natural infrared or ground station generated electromagnetic radio frequency signals. At present, Earth sensors meeting the required accuracy are in development stages but have not been proven.

Specific star candidates readily usable for azimuth measurement are the sun for its brightness and Ursa Minor (Polaris) for its nearly invariant location with respect to the orbital reference frame. Any pair of stars can be measured for attitude determination with star trackers, scanner or epoch comparator sensors. Most star measuring devices can exceed communication satellite attitude accuracy needs but require complex processing or mechanical servos to relate the attitude signals to an Earth-referenced system.

In all sensor systems, the life requirement is a serious design problem. Although present technology is such that there does not exist an off-the-shelf sensor meeting a 0.1° beam pointing accuracy, sensors of the future will undoubtedly fulfill this requirement. These future sensors will probably operate on basic principles considered at present, but will have increased sophistication in signal processing to increase the performance and design improvements to increase the sensor life. A detailed discussion of attitude sensors is beyond the scope of this paper. A brief discussion of characteristics of promising sensor systems is presented.

Horizon sensors measure pitch and roll error relative to the local vertical. The attitude information is dependent on the ability to detect the horizon radiance profile. Attitude determination accuracy of 0.05° or better, due to horizon uncertainty, should be attainable.[6] Other errors associated with the sensor could potentially be controlled so that the total sensor error is 0.07° or less. Present horizon signal processing techniques do not achieve this accuracy. More sophisticated detection and measurement schemes are being proposed which are thought to be capable of meeting 0.07° accuracy or less. Some achievements in accuracy might be obtained if effects due to seasonal changes and latitude and longitude could be incorporated in the signal processing. The effects could be processed on the ground and sent to the satellite as needed. The ground computational requirement would probably be less than that presently required for processing sensor data on spinning satellites to determine spacecraft attitude. The achievable accuracy could potentially approach 1 arcmin. In addition to accuracy improvements, techniques are needed to extend the life. Typically, present sensor design lifetime requirements are on the order of one year or less. To increase reliability, sensors will have a minimum of moving parts, although most sensors need some scanning device to achieve high accuracy. Static radiation balance devices can achieve high accuracy without moving parts.

An Earth-referenced attitude signal can be obtained from either an interferometer or monopulse sensor. Both are rf devices and have potential attitude accuracies of well under 0.1°. The systems are not currently in an advanced state of development for spacecraft use. Typically, rf systems are heavy (30–40 lb) and use a significant amount of power (30–50 w). The rf systems generally operate at high frequencies with a narrow pencil beam and need some other sensing system to get the satellite stabilized to within the beam width. Due to the baseline separation and multiple antennas needed for an interferometer system, a monopulse system is to be preferred to an interferometer system. A single ground station can provide two-axis, e.g., pitch and roll, information with either interferometer or monopulse. Yaw sensing can be achieved from detection of radio frequency polarization. Ground transmission of two equal and orthogonally polarized signals can be used for yaw angle determination. These signals can be separately transmitted or sent in rapid sequence. The yaw attitude reference plane bisects the angle formed by the transmittal polarization, and is properly aligned when the signal levels of two transmissions are equal and produce a zero difference or null. The principal error is Faraday rotation through the atmosphere which is 0.1° at 10 GHz and decreases inversely with the square of the frequency.

An rf device has a certain attractiveness due to the fact that a communications satellite will always be in contact with a ground station and will already have communication equipment on board, some of which might be compatible with the requirements for a monopulse. In the beam pointing mode, the data rate needed for attitude stabilization will be so low as to be negligible with respect to normal communication rates.

Star tracking devices are capable of high precision, 0.02° or less. One device of special interest is a Polaris tracker. In a direct control loop, i.e., no inertial reference, roll and yaw attitude information could be obtained to an accuracy of 0.1° or better after compensation for the deviation from true North. The projected weight and power of a Polaris tracker is approximately 10 lb and 10 w, respectively. Polaris tracker systems are in an early stage of development. System tests, experience, and an analysis are needed to verify 10-yr life capability.

Azimuth or yaw references can be generated from instrument gyros, updated and corrected for drift with vertical sensor data through gyrocompassing[7] or from star sighting. Instrument gyros are not considered serious candidates due to accuracy and life considerations.

Inertial-referenced star systems such as the Space Precision Attitude Reference System[8] are being developed and could have accuracies on the order of arcsecs. But this precise attitude information exceeds communication satellite requirements. The information must be processed in a computer to relate it to an Earth-referenced coordinate system. As in the case of the Polaris tracker, design verification is needed to prove the 10-yr life capability.

Sun sensors capable of 1 arcmin accuracy are being developed. They are probably best suited for use as a yaw reference during station keeping in the systems which do not have a yaw sensor. They would not be suitable for a full-time attitude reference as there are times when the sun line approaches the local vertical and there are periods of solar eclipse during certain periods of the year.

Summary of Wheel Systems

The study has indicated that a three axis sensing and control system with three reaction wheels is potentially lighter than a two-axis sensing and momentum stabilized control system. The weight advantage of the reaction wheel system increases with the severity of the required yaw accuracy. The power required for the two classes of system is essentially the same.

The primary differences in the two axis sensing systems is in the mechanism used to stabilize yaw. The one-wheel with thruster system uses the yaw component of control torque due to the offset thrusters to damp the yaw motion. Yaw restoring torque is provided by the momentum wheel. The single gimballed wheel and two-wheel systems depend on a desaturation mechanism to stabilize yaw. The desaturation mechanism also provides the restoring torque to counteract the inertial fixed component of solar torque, whereas the other momentum wheel systems use the gyroscopic stiffness of the wheel. The double gimballed wheel operates similar to the one-wheel system in coupling a component of roll signal to drive the yaw gimbal. Due to the control mechanization, the double gimballed wheel system does not exhibit a nutation motion or have a significant yaw overshoot due to roll transients. As a result of its improved dynamic performance, the double gimballed wheel system is preferred over the other momentum wheel candidates.

The recommended attitude control system without direct yaw sensing is the double gimballed wheel used as a momentum wheel system. With azimuth measurement, a three-axis control system is recommended. A Polaris tracker is a critical development item for the three-axis control system. Initial flight testing of a Polaris tracker might be accomplished in conjunction with a double gimballed wheel driven by a three-axis control law. In the event of Polaris tracker failure, the control law could be switched and the double gimballed wheel used as a momentum wheel system. After a long-life Polaris tracker has been flight-proved, the recommended system would be the three-axis reaction wheel system. It is anticipated that the three reaction wheels would be lighter than the double gimballed wheel.

The accuracy of all wheel systems is fundamentally sensor-limited, as indicated by Fig. 7. Projected improvements in Earth-referenced sensor accuracy currently under development meet 0.1° pointing accuracy. The two types of vertical sensing systems recommended for further development are the rf monopulse system and the horizon sensor system. The monopulse system is potentially the most accurate type of vertical sensor, but it requires a cooperative ground station at all times. The horizon sensor system operates autonomously. The choice of which of these two vertical sensing systems to use is dependent on total mission requirements.

Three-axis attitude control systems require a yaw sensor in addition to a local vertical sensor. Two types of yaw sensors are recommended for further development. They are Polaris trackers and rf polarization sensors. The Polaris tracker is potentially the most accurate yaw sensor and is currently under active development. A Polaris tracker is planned for use in the NASA Applications Technology Satellites F and G to be launched about 1972. The principal advantages of Polaris trackers are their inherent accuracy and autonomous operation. Advanced rf Polarization techniques are also under current development and limited flight experience has been recorded. The potential accuracy of rf polarization sensing systems is limited and rf techniques require an active ground station for operation.

System configuration and component selection must be based on system analysis of the entire mission requirements and on the state of equipment development at the time a spacecraft is built. Based on present development schedules, long-life, three-axis stabilized missions in the early 1970's are likely to be momentum wheel systems, when peak yaw torques are under 10^{-4} ft-lb. With the development of a long-life yaw sensor, e.g., a Polaris tracker, a three-wheel system is preferred for lower total system weight and higher accuracy in larger disturbance environments.

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